F.3 First Law of Thermodynamics

The first law is simply a generalization of the work-energy equation to include the additional internal energy we discussed, and the additional heat transfer method as well.

$$W + Q = \Delta E_{\text{mechanical}} + \Delta E_{\text{internal}}$$

$$W = \int_{V_1}^{V_2} P dV$$

$$\frac{dQ}{dt} = \begin{cases} kA\dfrac{\Delta T}{\Delta x} & \text{conduction} \\ hA\Delta T & \text{convection} \\ \sigma\varepsilon_{th} A\Delta T^4 & \text{thermal radiation} \\ \varepsilon_{sol} IA_{\perp} & \text{solar radiation} \\ \dfrac{dm}{dt} L_v & \text{evaporation} \end{cases}$$

$$E_{mech.} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy + \frac{1}{2}kx^2$$

$$\Delta E_{\substack{\text{internal} \\ \text{(neglecting} \\ \text{V dependence)}}} = \begin{cases} \int_{T_1}^{T_2} mcdT & \text{T change} \\ mL & \text{phase change} \end{cases}$$

$$E_{ideal\,gas}(T,V,N) = \frac{f}{2}Nk_BT$$

$$E_{Van\,der\,Waals\,gas}(T,V,N) = \frac{f}{2}Nk_BT - \frac{aN^2}{V}$$

$$E_{ideal\,solid}(T,V,N) = \frac{f}{2}Nk_BT + \frac{1}{2}\frac{B}{\upsilon}(V-\upsilon)^2$$

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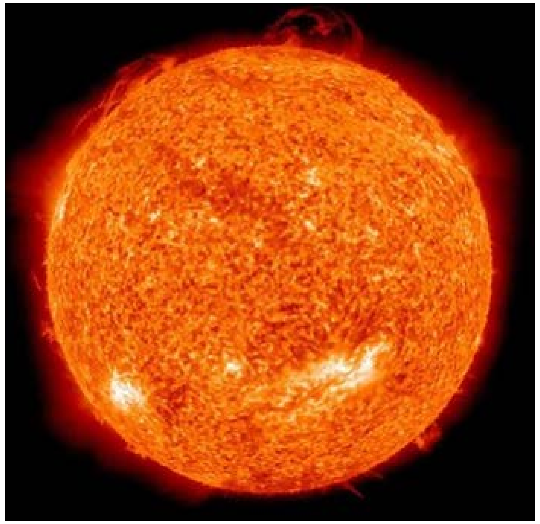
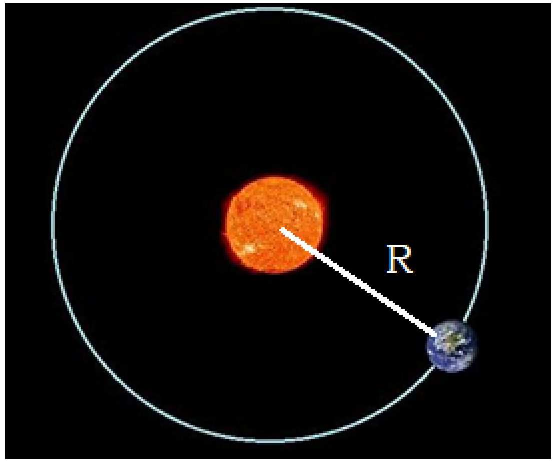


Temperature of Sun's surface is 5800K. And radius is about 700,000km. What is its power output? You can take ε_{th} = 1.

$$\frac{dQ}{dt} = \sigma\varepsilon_{th}AT^4$$

$$= \left(5.67\times10^{-8}\frac{W}{m^2K^4}\right)(1)\left(4\pi(700{,}000\times10^3\,\mathrm{m})^2\right)(5800\mathrm{K})^4$$

$$= 4\times10^{26}\,\mathrm{W}$$



What is the intensity of solar radiation coming Earth's way, a distance 1.5×10^{11}m away?

$$P = I\cdot Area \qquad \text{(from way back when)}$$

$$4\times10^{26}\,\mathrm{W} = I\cdot 4\pi R^2$$

$$4\times10^{26}\,\mathrm{W} = I\cdot 4\pi(1.5\times10^{11}\,\mathrm{m})^2$$

$$I = \frac{4\times10^{26}\,\mathrm{W}}{4\pi(1.5\times10^{11}\,\mathrm{m})^2} = 1400\mathrm{W}$$

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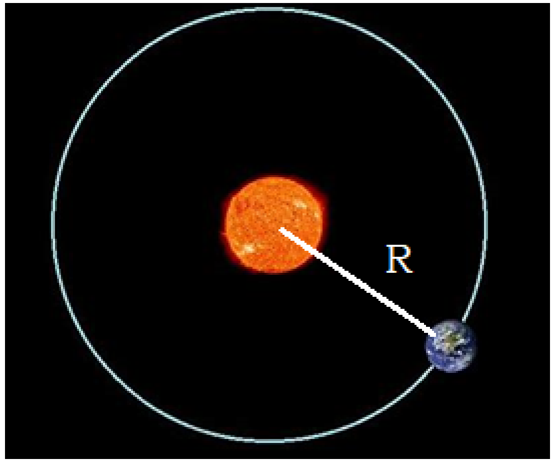

Suppose Earth reflects about 20% of this radiation, and absorbs the rest.
Earth also radiates energy into space, like the Sun does, by virtue of its temperature T, and absorbs energy from space, by virtue of its 2.7K temperature.

Supposing Earth radiates this energy from all points on its surface, and that ε_{th} = 1 for it, to what temperature would Earth equilibrate, accounting for just the sun's incoming radiation and the Earth's outgoing radiation?

$$Q + W = \Delta E_{mech.} + \Delta E_{\text{int.}}$$

$$(Q_{solar} + Q_{thermal}) + 0 = 0 + 0$$

$$\varepsilon_{sol} I A_{\perp} \cdot \Delta t + \sigma \varepsilon_{th.} A \Delta T^4 \cdot \Delta t = 0$$

$$(0.80)(1400)(\pi R^2)\Delta t + (5.67\times10^{-8})(1)(4\pi R^2)(2.7^4 - T^4)\Delta t = 0$$

$$(0.80)(1400) + (5.67\times10^{-8})(1)(4)(2.7^4 - T^4) = 0$$

$$2.7^4 - T^4 = -\frac{(0.80)(1400)}{(5.67\times10^{-8})(1)(4)}$$

$$T^4 = 2.7^4 + \frac{(0.80)(1400)}{(5.67\times10^{-8})(1)(4)}$$

$$= \left[2.7^4 + \frac{(0.80)(1400)}{(5.67\times10^{-8})(1)(4)}\right]^{1/4}$$

$$= 265^{\circ}\,\mathrm{K}$$

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If your body surface area is about 2m^2, and your temperature is 38C, at what rate do you radiate heat?

$$\frac{dQ}{dt} = \sigma\varepsilon A T^4 = (5.67\times10^{-8})(1)(2)(272+38)^4 = 1.05\text{kW}$$

Say you live in a semi-spherical igloo with radius r = 2m, and icey walls x meters thick. The outside temperature is -10C, and the inside temperature you want to maintain at 20C. The conductivity of ice is k = 2.5W/mK. What must x be?

We need the air inside to maintain its temperature at 20C. So the rate of heat from thermal radiation must equal the rate of loss due to conduction. We need.

$$Q + W = \Delta E_{mech.} + \Delta E_{\text{int.}}$$

$$(Q_{thermal} + Q_{conduction}) + 0 = 0 + 0$$

$$(1050\text{W})(\Delta t) + \left(kA\frac{\Delta T}{\Delta x} \right)(\Delta t) = 0$$

$$(1050\text{W}) + \left(2.5\frac{\text{W}}{\text{m}\cdot\text{K}} \cdot 2\pi(2\text{m})^2 \frac{-30\text{K}}{\Delta x} \right) = 0$$

$$\Delta x = \frac{2.5\cdot 2\pi(2)^2 30}{1050} = 1.8\text{m}$$